

O Level E-Math Cheat Sheet

Numbers

Prime Numbers	Numbers greater than 1 with only 2 factors - 1 and itself
Composite Numbers	Numbers with more than 2 factors
Rational Numbers	Numbers which can be expressed as a fraction -Inclusive of recurring decimals and whole numbers
Irrational Numbers	Numbers which cannot be expressed as a fraction Eg. π , $\sqrt{2}$
Prime Factorisation	Expressing a number as the product of its prime factors
HCF	Take the product of the lowest common power of prime factors
LCM	Take the product of the highest power of each prime factor

Standard Form

$$A \times 10^n$$

where n is an integer and $1 \leq A < 10$

Percentage

A fraction with 100 as the denominator:
Eg. $50\% = \frac{50}{100}$

Percentage change

$$\frac{\text{new} - \text{old}}{\text{old}} \times 100\%$$

Decrease = negative %
Increase = positive %

Inequalities

$\leq$ Less than or equal to
 $<$ Less than
 $\geq$ More than or equal to
 $>$ More than

*REMEMBER to flip the inequality sign when multiplying or dividing by a **negative** number

Approximation to 3s.f.

All final answers to be 3s.f. unless otherwise stated

(except angles which are to 1d.p.)

EXAMPLES OF 3s.f. ROUNDING

$$0.09876 \approx 0.0988$$

$$1.1234 \approx 1.12$$

$$12345 \approx 12300$$

Direct and Inverse

Proportion

Direct Proportion $y = kx$

Inverse Proportion $y = \frac{k}{x}$

where k is a constant

General guide to proportion qns

1. Form equation using formula above
2. Sub values for x and y to solve for k
3. Form new equation with k and new value of x or y subbed in
4. Solve for unknown

Map Scale

Always represented as 1:n

*Make sure both sides are **based on** the same unit

*Do not include units in scale

Area ratio

Square length ratio on both sides

Eg. Length ratio = 1 : n

$$\text{Area ratio} = 1^2 : n^2$$

Unit Conversions

$$1m = 100cm$$

$$1m^2 = (100)^2 cm^2 = 10,000cm^2$$

$$1km = 1000m$$

$$1km^2 = (1000)^2 m^2 = 1,000,000m^2$$

Indices

$$1. a^m \times a^n = a^{m+n}$$

$$2. a^m \div a^n = a^{m-n}$$

$$3. a^m \times b^m = (a \times b)^m$$

$$4. a^m \div a^n = \left(\frac{a}{b}\right)^m$$

$$5. a^{-m} = \frac{1}{a^m}$$

$$6. a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$7. a^0 = 1$$

Answers to be expressed in **positive indices unless otherwise stated

Hire-Purchase

Total Hire Purchase Amount = Total installments + Deposit

Interest = Total Hire Purchase Amount - Cash Price

Interest

Simple Interest

$$I = \frac{P \times r \times t}{100}$$

where

I = Interest

P = Principal Amount

r: **Annual** rate

t: Number of years

Compound Interest

$$A = P \left(1 + \frac{r}{100}\right)^n$$

where

A = Amount after compounding (P + I)

P = Principal Amount

r: Interest rate **per compounding period**

n: Number of times compounded

Eg. 24% **Annual** interest, compounded **monthly** over a period of **1 year**

r = Interest rate per month

$$= 24/12 = 2$$

$$n = 12$$

Set Notation

A $\cup$ B: Union of A and B

A $\cap$ B: Intersection of A and B

$\in / \notin$: is/ is not an element of

n(A): number of elements in set A

A': Complement of Set A

$\emptyset$ or $\{\}$: Null/ empty set

A $\subseteq$ B: Set A is a subset of Set B

*Every element of A is an element of B, where **A can be equal to B**

A $\subset$ B: Set A is a proper subset of Set B

*Every element of A is an element of B, where **A $\neq$ B**

Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

Number Patterns

Common Difference

3,5,7,9...

$$\text{General Term: } T_n = a + (n - 1)d$$

Common Ratio

2,4,8,16...

$$\text{General Term: } T_n = ar^{n-1}$$

where

a = first term

d = common difference

r = common ratio

Shortcuts!

Factorised form $(x - a)(x - b) = y$

x-intercepts = $(a, 0)$ and $(b, 0)$

Turning point:

$$\text{x-coordinate} = \frac{a + b}{2}$$

$$\text{y-coordinate} = \text{sub } x = \frac{a + b}{2} \text{ to find } y$$

Completed square form $y = \pm(x - h)^2 + k$

$$\text{Turning point} = (h, k)$$

Sketching quadratic graphs

Steps

1. Determine shape of curve
(positive x^2 = smile, negative x^2 = frown)
2. Find y-intercept by subbing x=0
3. Find x-intercepts (if any) by subbing y=0
4. Find turning point
5. Join points to form a symmetrical curve

Solving quadratic equations

Eg. $2x^2 - 4x - 6 = 0$

Method 1: Factorisation

1. Factorise to form: $(2x + 2)(x - 3) = 0$

2. Solve: $(2x + 2) = 0$ **or** $(x - 3) = 0$
 $x = -1$ $x = 3$

Method 2: Completing the square

1. Make coefficient of x = 1: $2(x^2 - 2x - 3) = 0$

2. Divide x^2 term by x $\frac{x^2}{x} = x$ $\frac{2x}{2x} = 1$

3. Divide x term by 2x

4. Subtract square of result from step 3

$$2(x^2 - 2x - 3) = 2((x - 1)^2 - 1^2 - 3)$$
$$= 2(x - 1)^2 - 8$$

Solve for x: $2(x - 1)^2 - 8 = 0$

$$(x - 1)^2 = 4$$

$$x - 1 = \pm\sqrt{4}$$

$$x = -1 \quad \text{or} \quad x = 3$$

Method 3: Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for } ax^2 + bx + c = 0$$

a= 2, b=-4, c=-6: $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$

$$x = -1 \quad \text{or} \quad x = 3$$

Radian

2π rad = 360 degrees

Arc length

Degree: $\frac{x^{\circ}}{360^{\circ}} \times 2\pi r$

Radian: $r\theta$

Sector Area

Degree: $\frac{x^{\circ}}{360^{\circ}} \times \pi r^2$

Radian: $\frac{1}{2}r^2\theta$

Segment Area

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta$$

where
 x° is the angle in degrees
 θ is the angle in radians

Area/ Volume of Similar Figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Statistics

Mean: Calculated Average
 Mode: Most frequent data value
 Median: Middle value of the entire range
 Lower/ Upper Quartile: 25th/ 75th percentile
 Interquartile range = Upper Quartile - Lower Quartile

Mensuration

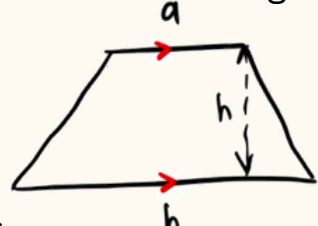
Parallelogram

Area
 = Base x Perpendicular Height



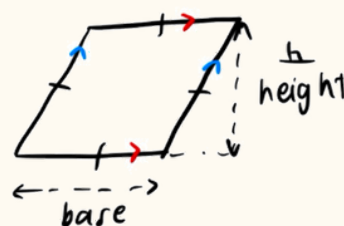
Trapezium

Area
 = $\frac{1}{2} \times (a+b) \times \text{height}$



Rhombus

Area
 = Base x Perpendicular Height



Pyramid/ Cone

Volume
 = $\frac{1}{3} \times \text{Base Area} \times \text{Height}$
 Surface Area
 = Curved SA + Base Area
 = $\pi r l + \pi r^2$



Sphere

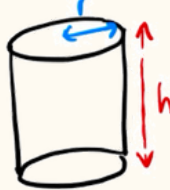
Volume = $\frac{4}{3} \times \pi r^3$
 Surface Area = $4\pi r^2$

Hemisphere

Volume = $\frac{2}{3} \times \pi r^3$
 Curved Surface Area = $2\pi r^2$

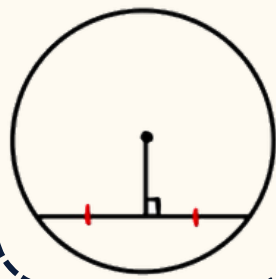
Prism/ Cylinder

Volume
 = Base Area x Height
 Eg. Cylinder volume = $\pi r^2 h$

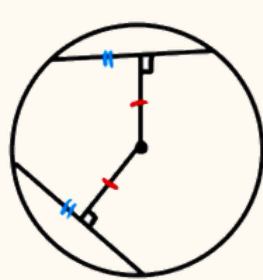


Properties of Circles

$\perp$ bisector of chord



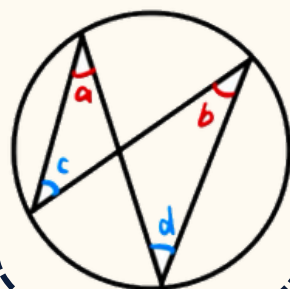
2 equal chords are equidistant from centre



Angles in the same segment

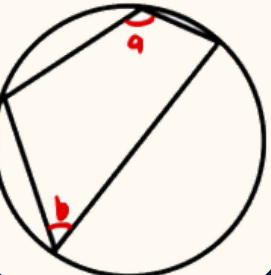
$$\angle a = \angle b$$

$$\angle c = \angle d$$



Angles in opp. segments

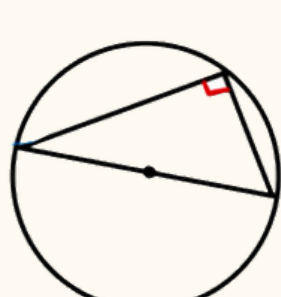
$$\angle a + \angle b = 180^{\circ}$$



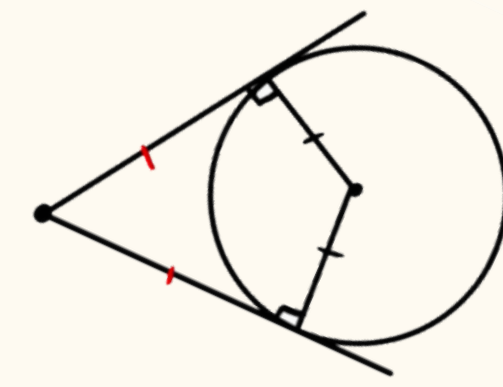
Tan $\perp$ Rad



Right angle in semicircle

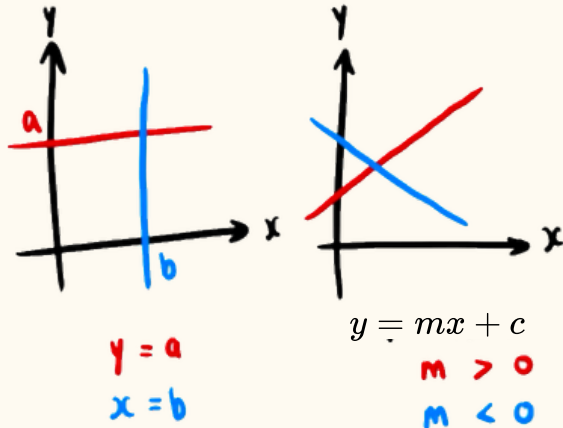


Tangents from external point

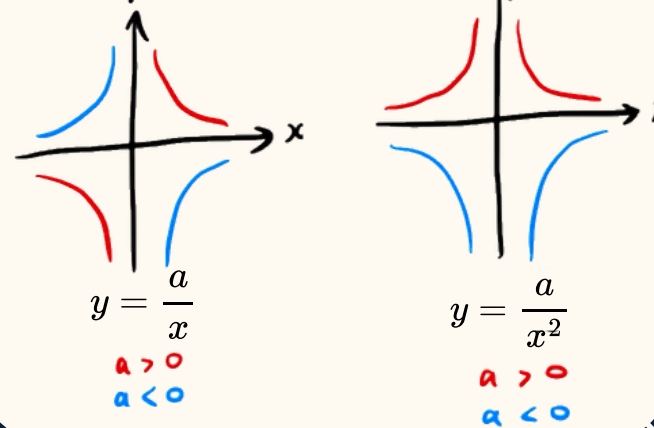


Graphs

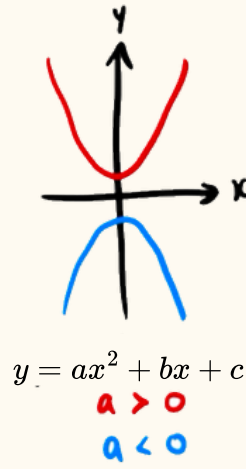
Linear



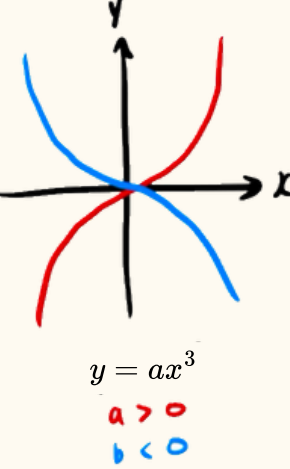
Reciprocal



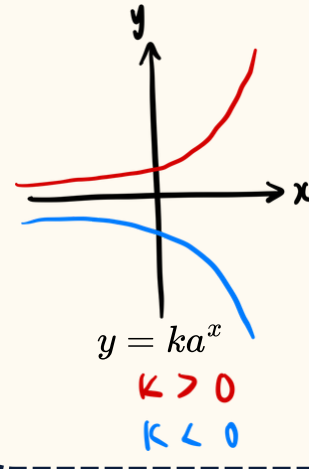
Quadratic



Cubic



Exponential



Congruence Tests

- SSS test
- SAS test (Angle must be in between 2 sides)
- AAS test (Side must be in between the 2 angles)
- RHS test (Right angle, Hypotenuse, Side - for $\triangle$ triangles)

*A: Corresponding angles are equal

S: Corresponding sides are **equal**

Similarity Tests

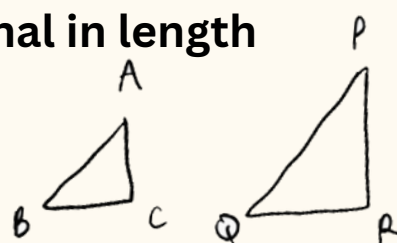
- AA test
- SSS test
- ASA test (Angle must be in between the 2 sides)

*A: Corresponding angles are **equal**

S: Corresponding sides are **proportional in length**

Eg.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$



Coordinate Geometry

For the equation of a straight line: $y=mx+c$

m: gradient

c: y-intercept

Distance between 2 points=

$$\text{gradient} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

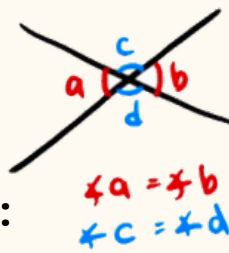
Angles

Types of angles

- Acute angle: $0^{\circ} < \theta \leq 90^{\circ}$
- Obtuse angle: $90^{\circ} < \theta \leq 180^{\circ}$
- Reflex angle: $180^{\circ} < \theta \leq 360^{\circ}$

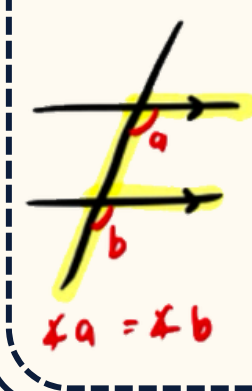
Common Angle Properties

- Sum of angles on a straight line = 180°
- Sum of angles at a point = 360°
- Vertically opposite angles:

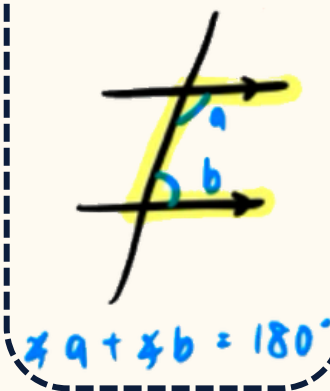


Parallel Lines Angle Properties:

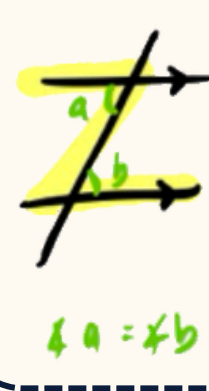
Corresponding Angles



Interior Angles



Alternate Angles



Properties of polygons

In a regular polygon with n sides,

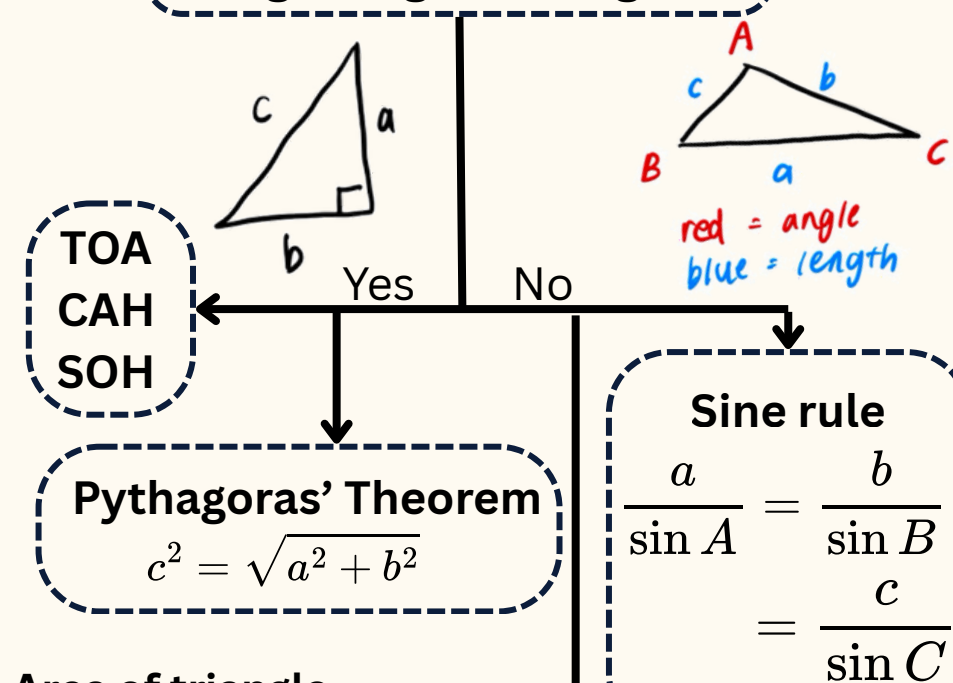
Each interior angle = $\frac{(n - 2) \times 180^{\circ}}{n}$

Each exterior angle = $\frac{360^{\circ}}{n}$

*Interior angle + exterior angle = 360°

Trigonometry

Right angled triangle?



Area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} ab \sin C$$

Bearings:

- Measured from the North in a clockwise direction
- Represented by a 3 figure number (and to 1d.p if needed)
 Eg. 12.34° will be represented as 012.3°

Vectors

Position Vector of A = $\vec{OA}$

If $\vec{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$, coordinates of A = (x, y)

Magnitude (Length) of vector $\begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{x^2 + y^2}$

Finding a vector from its position vectors:

$$\vec{AB} = \vec{OB} - \vec{OA}$$